

2.3/29 | Let $f(x) = \begin{cases} 2x^2 + 5, & x < 0 \\ \frac{3 - 5x^3}{1 + 4x + x^3}, & x \geq 0 \end{cases}$

a) $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow -\infty} 2x^2 + 5 = +\infty$

"decreases without bound"

b) $\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} \frac{3 - 5x^3}{1 + 4x + x^3} = -5$

$\Rightarrow \lim_{x \rightarrow \infty} \frac{x^3 \left(\frac{3}{x^3} - 5 \right)}{x^3 \left(\frac{1}{x^3} + \frac{4}{x^2} + 1 \right)}$

$= \lim_{x \rightarrow \infty} \frac{\frac{3}{x^3} - 5}{\frac{1}{x^3} + \frac{4}{x^2} + 1} = \frac{-5}{1} = -5$

$$f(x) = \frac{x+5}{x+6}$$

$$g(x) = f(x) \cdot (\text{---})$$

$\lim_{x \rightarrow 0} g(x)$ is indeterminate

$$\lim_{x \rightarrow \infty} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

$$\lim_{x \rightarrow 0}$$

$$\frac{x^2+5x}{x^2+6x}, \frac{\frac{1}{x}+\frac{5}{x^2}}{\frac{1}{x}+\frac{6}{x^2}}, \frac{x(1+\frac{5}{x})}{x(1+\frac{6}{x})}, \frac{x^3+5x^2}{x^3+6x^2}, \dots$$

2.2/31)
a) $\lim_{x \rightarrow 0^+} \left(\frac{1}{x} - \frac{1}{x^2} \right) = \lim_{x \rightarrow 0^+} \frac{1}{x} - \lim_{x \rightarrow 0^+} \frac{1}{x^2}$

$= \boxed{+\infty - (+\infty)} = 0$

can not be done
(individual limits don't exist)

p.123

2.2)
37b

$$\lim_{x \rightarrow 0^+} \left(\frac{\cancel{x} \left(\frac{1}{\cancel{x}} \right) - \frac{1}{x^2}} \right) =$$

$$\frac{x}{x^2} - \frac{1}{x^2} = \frac{x-1}{x^2}$$

$$\lim_{x \rightarrow 0^+} \frac{x-1}{x^2}$$

$$\left(\frac{-1}{0} \right) \rightarrow -\infty$$

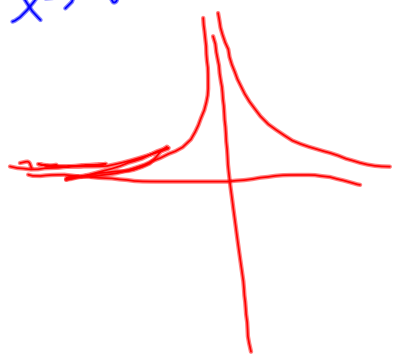
2.3/23

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^4 + x}}{x^2 - 8} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^4(3 + \frac{1}{x^3})}}{x^2(1 - \frac{8}{x^2})}$$

$$= \lim_{x \rightarrow -\infty} \frac{\sqrt{x^4} \sqrt{3 + \frac{1}{x^3}}}{x^2(1 - \frac{8}{x^2})} = \lim_{x \rightarrow -\infty} \frac{\sqrt{3 + \frac{1}{x^3}}}{1 - \frac{8}{x^2}} = \frac{\sqrt{3}}{1}$$

$\frac{x^1}{x^4} = x^{1-4}$
 $= x^{-3}$
 $= \frac{1}{x^3}$

$3x^4 + x =$
 $x^4 \left(\frac{3x^4}{x^4} + \frac{x^1}{x^4} \right)$



$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 + x}}{x - 8} = \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 \left(3 + \frac{1}{x}\right)}}{x \left(1 - \frac{8}{x}\right)}$$

But —
this is
always
NEGATIVE

for every $x < -\frac{1}{3}$

$$= \frac{\sqrt{x^2} \sqrt{3 + \frac{1}{x}}}{x \left(1 - \frac{8}{x}\right)} = \frac{+ \sqrt{3}}{1 - \frac{8}{x}}$$